

Visual divergence in humans and computers



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Studies of design creativity have underlined the importance of divergent reasoning and visual reasoning in idea generation. Connecting these two key design skills, this paper presents a model of divergent visual reasoning for the study of creativity. A visual divergence task called ShapeStorm is demonstrated for the study of creative ideation that can be applied to humans as well as computational systems. The model is examined in a study with human subjects, a computational stochastic generator, and a geometrical analysis of the solution space. The main significance of this task is that it offers a straightforward means to define a simple design task that can be used across research studies. Several scenarios for the application of ShapeStorm for the study of creativity are advanced.

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Creative ideation is generally considered to involve divergent reasoning processes to generate original, useful and unexpected ideas (Maher, 2010; Schunn, Paulus, Cagan, & Wood, 2006). A majority of methods and techniques for idea generation prioritise quantity of ideas and suggest deferring evaluation; and research does suggest that more ideas generated in the early phases of the design process lead to more creative final outcomes (Yang, 2009). Divergent reasoning – the process of generating as many different alternatives as possible, is recognised as one core component of the design process. Visual reasoning has also been identified as an important function for design creativity (Goel, 1995; Kavakli & Gero, 2001; Shah, Millsap, Woodward, & Smith, 2012).

Divergent reasoning is usually assessed in linguistic rather than visual formats, and visual reasoning is often studied in relation to cognitive constructs such as perception, encoding and chunking, working and long-term memory, mental transformation and representation (Bilda & Gero, 2007; Shah et al. 2012). Connecting these two key design skills, this paper presents a model of divergent visual reasoning for the study of creativity. The aim of this work is to propose and demonstrate a design task or problem statement that researchers of

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design creativity can employ to study visual ideation. This type of assessment approach can be applied to humans as well as computational design systems, thus providing a foundational step towards the comparative study of human and computational creativity.

At present, studies of design ideation lack explicit and objective criteria to define the task or problem statement used in models and experiments. It is common practice in design creativity research to define significantly different design tasks or challenges for participants to generate ideas. General models of design ideation based on visual tasks are needed which can systematically guide human and computational studies. In other fields, classic problem statements are extensively used across studies although not without criticisms and caveats, i.e., the classic ‘thumbs problem’ (Taylor, Berry, & Block, 1958), the Remote Associates Test (Mednick, 1968), and the ‘nine-dot puzzle’ (Burnham & Davis, 1969), among others. The study of specialised assessment tools for specific design skills is an active research topic. The work presented in this paper aims to contribute an approach to be applied across research areas of creativity.

The way in which tasks are defined today can be problematic, and more so in studies of design creativity. The main problems include: a) design tasks are arbitrarily chosen and framed by the researchers making it difficult to compare results across studies; b) problem statements are framed in ad-hoc ways often excluding key information or constraints; c) tasks may or may not be partially structured, open-ended, and many require domain knowledge or refer to problem situations inspired by real world conditions, making it impossible to objectively assess performance, leading to d) strong dependence on domain experts to judge early ideas often poorly communicated in elementary representations; and e) early ideas that are produced in short sessions are, by definition, incomplete and some may contain subcomponents or partial aspects of merit that judges can easily oversee or inadequately reinterpret during assessment. Due to such lack of consistency in the study of design creativity, the constructs being analysed, the relevance of the findings and the validity of the claims remain problematic. It is difficult enough to systematically study the generation and evaluation of creative ideas, even more so when no basic agreement exists across studies.

This paper presents *ShapeStorm*, a model of visual ideation that captures key characteristics of design activity: no unique correct solution, adaptable problem formulation, combination of objective and subjective evaluation criteria, and exploration of solution space by visual reasoning (Goel, 1995). In addition, *ShapeStorm* is suitable for the computational study of design creativity and innovation, and it is suitable for a range of experimental conditions with human subjects. In the remainder of the paper, Section 1 presents a background analysis of tasks used across creativity research highlighting current

practices of assessment approaches. Section 2 introduces *ShapeStorm* including the task description, an experimental study of hand-drawn solutions by human subjects, and an algorithmic implementation. Section 3 presents a geometric approach to comprehensively characterise the solution space. The paper closes with a general discussion and sample applications of this task in supporting our reasoning about creativity.

1 Background

In studies of design creativity, participants are customarily required to generate ideas in response to a given design task or problem. These early ideas are subsequently classified and often evaluated by the researchers, by judges trained by these, or by domain experts who draw conclusions about the process or the subjects across experimental conditions. Individuals are classified and ideation techniques and methods are validated using variations of this general approach. Robust assessment techniques are needed, starting with the way in which a task or design problem is defined in such studies. Design problems have been characterised as *wicked* or ill-formulated, indeterminate, with no implicit stopping rules, where solutions cannot be true or false, with no exhaustive list of admissible operations, and where defining and framing the design problem is seen as a key aspect of creativity (Dorst & Cross, 2001; Goel, 1995; Rittel & Webber, 1973). In this paper the term *design task* is used rather than *design problem* to avoid the implication that ‘the design involves addressing problems that exist as separate entities, like puzzles that have predetermined correct answers’ (Herbert, 1992). In the following sections, design tasks from creativity research and from computational creativity are analysed as a way to frame a model of visual divergence that can be applied in both research fields.

1.1 Design tasks in creativity research

An analysis of 32 design tasks used in studies of design creativity is summarised here (Becattini, Borgianni, Cascini, & Rotini, 2012; Benami & Jin, 2002; Cardoso & Badke-Schaub, 2009; Casakin & Kreidler, 2011; Daly, Yilmaz, Christian, Seifert, & Gonzalez, 2012; Ellamil, Dobson, Beeman, & Christoff, 2012; Fu et al. 2012; Genco, Holttä-Otto, & Seepersad, 2012; Gero, Jiang, & Williams, 2013; Goel & Grafman, 2000; Goldschmidt & Sever, 2011; Hernandez, Shah, & Smith, 2010; Kershaw, Hölttä-otto, & Lee, 2011; Kim, Lee, Kim, Park, & Jeong, 2011; Kowatari et al. 2009; Lotz et al. 2013; Mehta, Zhu, & Cheema, 2012; Mohan & Shah, 2012; Moreau & Dahl, 2005; Mougnot & Watanabe, 2012; Shah et al. 2012; Skulberg, 2012; Smith, Ward, & Schumacher, 1993; Suwa & Tversky, 2003; Thoring & Mueller, 2012; Viswanathan & Linsey, 2012; Wilson, Rosen, Nelson, & Yen, 2010; Worinkeng, Summers, & Joshi, 2013; Yilmaz, Seifert, & Gonzalez, 2010; Yu & Nickerson, 2011; Yuan, Chiu, Lee, & Wu, 2012; Zeng, Proctor, & Salvendy, 2011). This collection gives an indication of the diversity of design tasks used in the field,

and their selection is illustrative rather than exhaustive. The design tasks were extracted from recent or prominent experimental research; future work will focus on a systematic meta-analysis of such studies. The main observations include:

- 1) High variance in the level of specificity, from a short sentence to a few paragraphs with detailed requirements and constraints, i.e.,:
 - a) ‘Design two products: a chair for children and a desk clock’ (Goldschmidt & Sever, 2011); ‘Please design a chair for children’ (Yu & Nickerson, 2011); ‘Design a next-generation alarm clock to be sold in the contemporary marketplace’ (Genco et al. 2012); ‘Design a new pen’ (Kowatari et al. 2009).
 - b) ‘We approximated a real-world architectural planning task by requiring the patient to develop a new design proposal for our lab space: Our lab space is located in Room 5D51. It currently houses three scientists and five research assistants. Another scientist is expected in January. The number of research assistants can increase up to 16 during the summer months. The space is used for reading, writing, computing, telephone conversations, and so on. In fact, we do all of our work in this space except for seeing patients. Some of us spend up to 10 h per day there. It is a very dismal environment. Your task is to reorganise, and redesign the space such as to increase our comfort and productivity. We do not have a budget for the redesign. However, we do have the option of exchanging some of our furniture at the surplus store, and perhaps we can pool personal time and resources to do some painting and cleaning. You have 2 h to propose a design. You may spend up to 15 min of the first hour in the lab space. While there, you may measure, make notes and sketches, and ask anyone there any questions you think relevant. You may revisit the lab for 10 min anytime during the second hour. Please begin.’ (Goel & Grafman, 2000); ‘Design a device to transport a ping-pong ball. The device should be powered by only a spring. The objective is to travel the farthest horizontal distance, measured perpendicular to the starting line. Operation: You are not allowed to have any contact with the device when in operation. Time is not a factor; only the distance will be measured after the device comes to stop. Regulations: a) You can only use the materials listed below. No other material may be used or substituted outside what has been specified; b) You can cut and deform any of these materials in any way you like; c) You can use adhesives, staples, scotch tape, and solder to make the joints of the structure’ (Hernandez et al. 2010; Shah et al. 2012).
- 2) High variance in themes, domains, functions, target users and levels of reality and complexity: design concepts for everyday appliances (Gero et al. 2013; Mougnot & Watanabe, 2012); medical devices (Wilson et al. 2010); mechanical devices with specific functional requirements (Fu et al. 2012;

Mohan & Shah, 2012; Worinkeng et al. 2013); and toys (Moreau & Dahl, 2005; Yilmaz et al. 2010).

- 3) High variance of time length allocated for the design task, going from a few seconds (Kowatari et al. 2009), a few minutes or hours (Casakin & Kreitler, 2011; Daly et al. 2012; Ellamil et al. 2012; Smith et al. 1993; Suwa & Tversky, 2003), to semester-long projects (Thoring & Mueller, 2012).
- 4) High variance on types and levels of data collection and analysis including conversations, gestures and drawing audio and video recorded (Gero et al. 2013), classification and evaluation of text descriptions and sketches (Fu et al. 2012), neural scanning (Kowatari et al. 2009), and interviews (Thoring & Mueller, 2012).
- 5) High variance of participant profiles, ranging from first year undergraduate design students (Skulberg, 2012) to final year (Cardoso & Badke-Schaub, 2009), engineering students (Worinkeng et al. 2013), psychology students (Yilmaz et al. 2010), practicing engineers and architects (Daly et al. 2012; Goel & Grafman, 2000), and general public (Suwa & Tversky, 2003; Yu & Nickerson, 2011).
- 6) High variance of idea assessment practices, resulting in the value of creativity being ascribed either by a panel of judges (ranging from those who setup the problem to assistants trained for the job, or consensual assessments by domain experts), by objective performance criteria defined in the task, or in between-group or within-subjects evaluations.

Whilst it could be argued that these studies target ideation processes that are independent from the task at hand, it is reasonable to question what type of claims about design creativity can be made based on ideas generated in such varying task types and conditions. Are participants expected to produce similar results or conduct consistent design processes whether they tackle a 5-min imagination exercise with no constraints, a 1 hr challenge with explicit performance metrics, or a semester project that includes background and user research, observation, ideation, prototyping, and feedback? Should researchers assume the early ideas ranked highest by the judging panels would necessarily lead to the most original and appropriate final solutions? These questions are relevant in view of the claims often made by researchers of design ideation. Whilst diversity in the study of design creativity may be a positive direction, the current disparities found in the literature indicate a lack of a minimal set of criteria to move the field forward.

1.2 Tasks in computational creativity research

Computational creativity refers to the design and implementation of systems that either seek to exhibit artificial creative behaviour or seek to support the reasoning about human creative behaviour. Such systems have been studied in recent years across fields including music, visual arts,

poetry, narrative, and various design areas (Cardoso, Veale, & Wiggins, 2009). Computational systems to study design creativity require the modelling of autonomous agents that generate solutions to a given problem, so the researchers can draw conclusions about the fundamental processes at work and the possible ways of triggering desired outcomes. Combining computational design synthesis and design creativity is a recent venture (Campbell & Shea, 2014).

An analysis of 22 tasks used in studies of computational creativity is summarised here (Cohen, 1999; Colton, 2008; Cook, Colton, & Gow, 2013; French & Hofstadter, 1991; Gabora & Kitto, 2013; Gero & Sosa, 2008; Hemberg et al. 2007; Hsiao & Chen, 1997; Jadhav, Joshi, & Pawar, 2012; Lewis & Parent, 2000; Machado & Pereira, 2012; McGraw & Hofstadter, 1993; McGreggor, Kunda, & Goel, 2010; Norton, Heath, & Ventura, 2013; Orsborn, Cagan, Pawlicki, & Smith, 2006; Pereira & Cardoso, 2006; Pérez y Pérez, Sosa, & Lemâitre, 2007; Ross, Ralph, & Zong, 2006; Saunders & Gero, 2004; Schnier & Gero, 1998; Sosa, 2005). Since we are interested in developing design tasks applicable in studies of humans and computational systems, comparing the ways in which these fields define the experimental settings to study creativity reveals important features to consider. This illustrative sample is drawn from computational tasks where visual representations are instrumental either to represent or to implement the system, thus excluding tasks from domains such as literature and music. A number of problem statements used in computational creativity appear to be defined mainly to demonstrate the feasibility or to benchmark the performance of a research approach (French & Hofstadter, 1991; Saunders & Gero, 2004; Sosa, 2005). However, other cases more clearly articulate problem-solving goals to generate novel solutions in a domain (Cohen, 1999; Colton, 2008; Gabora & Kitto, 2013; Jadhav et al. 2012).

The level of task complexity in computational cases is generally lower than in design cases as judged by the mean length and level of detail of the problem statements, with a vast majority consisting of a single sentence, i.e., ‘To create new and unique vehicles representing the language of design for coupes, pickups, and SUVs’ (Orsborn et al. 2006); ‘Goal is to blend two domains and interpret the newly generated instances according to an unambiguous process’ (Pereira & Cardoso, 2006); ‘Automatic synthesis of aesthetically pleasing images’ (Ross et al., 2006); ‘Design adaptive vehicle dashboard panels’ (Gero & Sosa, 2008).

High variance is observed in the themes covered in computational creativity, including: font design (McGraw & Hofstadter, 1993); architectural designs (Pereira & Cardoso, 2006); fine art (Cohen, 1999; Gabora & Kitto, 2013); product design (Hsiao & Chen, 1997; Orsborn et al. 2006); videogame dynamics (Cook et al. 2013); visual narratives (Perez et al. 2007); and

choreographic movements (Jadhav et al. 2012). A wide range of modelling techniques is seen across these models, and ad-hoc assessment practices make it difficult to compare measures of creativeness across the field.

Bridging design tasks across humans and computers is far from trivial. A comparison between the two sets shows that only a few tasks could potentially be used in experimental studies and implemented computationally, such as Becattini et al. (2012) where the problem could be represented in a closed system of fasteners and assemblies. In fact, McGreggor et al. (2010) present such a case with a computational system that solves a standardised IQ test used with humans – where a unique correct answer is required. More open-ended and qualitative cases are challenging, but work such as automated literary composition (Gervás, 2010) could suggest possible computational approaches combining linguistic and visual representations to tackle a subset of these tasks (Ellamil et al. 2012; Thoring & Mueller, 2012). On the other hand, evaluation in computational creativity is often done by the researcher, or alternatively by asking evaluators to distinguish human-made from artificial solutions. The authors could not find visual divergent tasks in the literature applied to human and computational systems, particularly tasks that are appropriate for the study of design problems, and that include equivalent assessment criteria for humans and artificial design agents.

1.3 Visual divergence tasks

Certain cases such as Yilmaz et al. (2010) and Moreau and Dahl (2005) that deal with shapes as initial inputs, can be related to studies of discovery of emergent structures in images (Finke, 1995). Such studies are based on visual divergence and have shown that subjects are capable of taking simple geometric forms, imagining new combinations, and then discovering emergent patterns and symbols that can be judged as creative by external observers. To that end, participants are given standard shapes such as those shown in Figure 1a and are instructed to mentally combine them without altering their essential form to produce interesting compositions. Sample resulting ‘preinventive forms’ are shown in Figure 1b. According to these studies of mental synthesis, most creative inventions are obtained when subjects do not define a target category until after they generate such preinventive forms, with more conventional designs resulting when a category is predefined in advance. The *Geneplore* (generate-explore) model (Finke, 1995) constraints are progressively incorporated, such as ‘the functions they must serve, or the particular categories to which they must belong’ (Finke, 1995).

In mathematical creativity, assessments of creative thinking ability include ‘multiple solution tasks’ (MSTs) (Leikin, 2009) where students are required to find a unique correct solution to a problem in as many different ways as possible. The solution spaces in MSTs distinguish between processes that

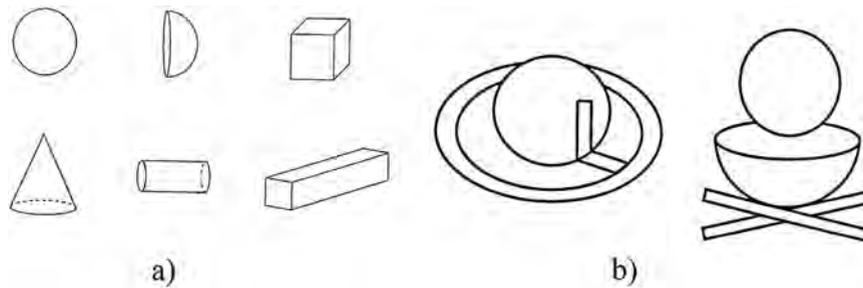


Figure 1 a) Standard 3D shapes and b) Sample preinventive forms as defined in Geneplore (Finke, 1995)

are generally recommended by the standard curriculum, displayed in textbooks, and usually taught by the teachers (less creative), from those based on strategies usually not prescribed by the school curriculum, or that the curriculum recommends with respect to a different type of problem. More relevant to divergent thinking, assessments based on ‘open-response’ problems that require more than one correct answer are infrequent, such as the arrangement of shapes along their sides (Kim, Cho, & Ahn, 2004).

The design task presented in this paper aims to contribute in the understanding of divergence across human and computational systems by providing a visual means to examine design creativity. Based on the current practices illustrated above, our strategy is to adopt a ‘small-scale system’ approach (Montfort & Fedorova, 2012) with design tasks of low complexity along the practice of Yilmaz et al. (2010), Moreau and Dahl (2005), Suwa and Tversky (2003), Ross et al. (2006), and Gero and Sosa (2008) until a deeper and more consistent understanding is built by switching back and forth between human and computational studies.

2 ShapeStorm model

The design task *ShapeStorm* originated in the practice and teaching of sketching as a tool for creative thinking in design. It can be defined verbally as: ‘Combine g geometries of s sides to generate as many unique shape compositions $\{g's\}$ resulting in more than g final geometries ($g' > g$). The initial geometries can be moved, rotated, and scaled but their number of sides must remain constant’. The uniqueness criterion is defined by the topological features of the resulting shape compositions as explained below. Such divergent task captures some of the open-ended aspects of design problems (Goel, 1995), it supports emergence since new shapes produced by the combination operation are not explicitly represented at input time, and it can be adapted or extended to include different types of constraints, and additional evaluation or fitness criteria. This paper presents an in-depth study of the simplest form of *ShapeStorm*: the combination of two three-sided shapes ($2g:3s$) which consists of {AND, XOR} Boolean operations (Stiny, 2008), namely:

$$(g_1, g_2) \rightarrow (g_1 - g_2, g_1 + g_2, g_2 - g_1) \quad (1)$$

The simplest case of *ShapeStorm* ($2g:3s$) produces results including those represented graphically in Figure 2. The following notation is used to represent shape compositions: $\{g', i, l, v, o, (s'_1 \dots s'_n)\}$ where: g' = number of final geometries, i = points located inside the area of the initial geometries, l = points located in the perimeter of the initial geometries, v = points coincident with a vertex of the initial geometries, and o = points located outside the area of the initial geometries; lastly, $(s'_1 \dots s'_n)$ is a subset of size g' that defines the sides for every final geometry g' .

The composition in Figure 2a is labelled as $\{3, 1, 0, 0, 5, (3, 4, 6)\}$, where the first digit indicates that three shapes are generated of 3, 4 and 6 sides respectively (shown between parentheses), and the sequence '1,0,0,5' indicates that one point is inside a geometry, no points are on edge or vertices, and five points lie outside the geometries. In Figure 2b, three final geometries are generated of 4, 5 and 5 sides – the change caused by two points now lying within the area of the initial geometries – or in i position. In Figure 2c and 2d five final geometries are generated, in 2c with all six points in o position and in 2d with four points in o position and 2 in l position – or in the edge of a geometry. The primary goal in *ShapeStorm* is to generate as many valid ($g' > g$) and unique shape compositions ($\{g', i, l, v, o, (s'_1 \dots s'_n)\}$) as possible; more criteria can be added to customise the task, such as to generate compositions with a specific number of sub-shapes, a target number of sides, or a specific set of topological features.

A variety of compositions can be generated based even on such basic version of *ShapeStorm* ($2g:3s$), and the number rapidly escalates as g and s increase. Notice that 'idea generation' in this task could consist of manipulating points, lines, or polygons in order to produce as many different outcomes as possible.

The resulting shape compositions produce not only other triangles but new classes of shapes of four and more sides, illustrating the generative capacity of this task to create emergent polygons that could then be used as inputs in

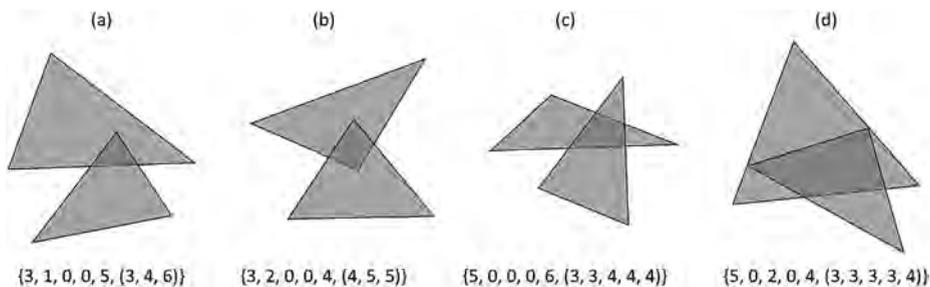


Figure 2 Four sample unique shape compositions built with two triangles and their notation

successive stages, thus bridging combinatorial and transformative creativity. This characteristic can be used to articulate and reason about theoretical debates such as the role of blind-variation and selective-retention in creative systems.

In *ShapeStorm*, ideation fluency (f) is estimated by the total output of valid and unique solutions generated, whilst originality (y) by the number of instances each solution is generated in a reference group. Composite (f,y) scores can be derived by researchers for instance considering the number of unique solutions generated by an individual, a weighted average, etc.

2.1 Hand-drawn ShapeStorm

ShapeStorm has been examined with hundreds of participants over the last years in pencil-and-paper sketching sessions, mainly with undergraduate and graduate engineering and architecture students. The learning objective of this activity has been to use basic hand drawing skills as a thinking aid in creative visual problem solving. It is typically run as a 15-min challenge after a 2-h learning module on rapid idea sketching. In addition to the instructions described above, one sample solution is customarily shown to further clarify the task and the notation system used to describe valid solutions (Figure 2a).

Participants engage in this activity individually and in silence, and they have provided generally positive feedback about tackling this visual thinking challenge. For the purpose of illustrating the type of results of *ShapeStorm* in this paper, the exercise is replicated here with 25 participants, all undergraduate students across disciplines (design, engineering, and business). Median age is 21 years, 15 male and 10 female. Participants sketch their solutions by hand in A3 sheets and after 15 min they help in the self-evaluation of solutions applying the notation previously described. These are double-checked by the researchers for validity and uniqueness.

Figure 3 shows a typical worksheet from a participant including invalid or duplicated combinations crossed out. Fluency and originality assessment of the set of solutions is described here (245 responses for an average of 9.8 solutions per individual). As presented earlier, assessments in *ShapeStorm* are objectively determined, thus avoiding biases by experts, researchers or audiences.

In the study presented here, high variance in fluency is observed ($f = 9.8$, standard deviation 3.01). Originality is estimated by weighing solutions by the total number of occurrences in the group, so for a solution provided by only one individual, $y = 1$, whereas for an entry provided by ten individuals $y = 0.1$. All y scores are added to obtain an individual's overall divergent indicator.

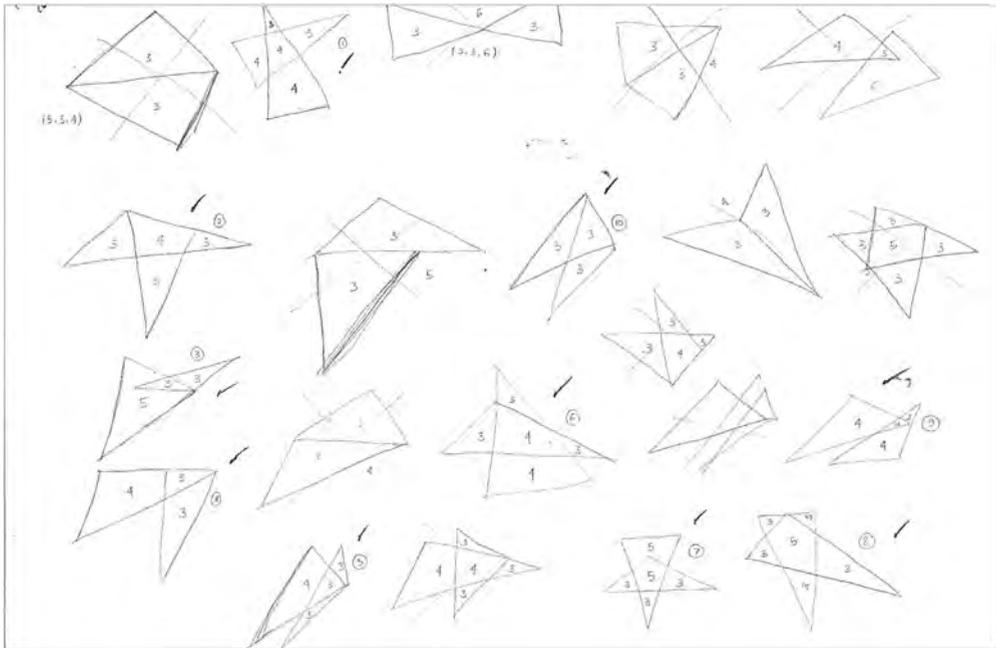


Figure 3 Sample hand-drawn ShapeStorm worksheet including invalid or duplicated combinations crossed out

The results of this exercise are shown in [Table 1](#): rows are the solutions generated by participants described by the notation g' and $(s'_1 \dots s'_n)$, previously defined and columns are all the participants, showing with a check mark the solutions generated by each individual. A total of 37 unique and valid solutions are generated collectively, with average fluency $f = 9.8$, $f_{min} = 5$ and $f_{max} = 16$. The solutions are ranked by originality (y) as shown in the last column; a total of 7 solutions were only generated once ($y = 1$), whereas the 7 more common solutions were generated by more than half of the participants comprising nearly 50% of all solutions: 117 entries out of the 245 compositions ($y \leq 0.07$). Future studies can investigate the factors associated with this clear difference between 'low-hanging fruit' solutions (easily accessible) and those that seem harder to reach. The performance of nominal and real groups can also be analysed, or the effects of priming subjects with less common solutions, or solutions with particular topological features.

Based on this initial study which yields results that are consistent with our experience running this task with hundreds of participants, if the results can be generalised, there seems at first to be a moderate correlation between fluency (f) and originality (y) ($R = 0.74$, $N = 25$, significant at $p < 0.05$), confirming the general tendency to use fluency as indicator of creativity in ideation studies. However, looking at the results in more detail shows that average values are not that meaningful in the study of creative activity: the two most prolific subjects in our study ($f = 16$) rank fourth ($y = 2.5$) and sixth

Table 1 Results of hand-drawn *ShapeStorm* with 25 participants: rows are solutions generated and columns are individuals, check marks show the solutions generated by each individual

| G' | ($s'1 \dots s'n$) | $s1$ | $s2$ | $s3$ | $s4$ | $s5$ | $s6$ | $s7$ | $s8$ | $s9$ | $s10$ | $s11$ | $s12$ | $s13$ | $s14$ | $s15$ | $s16$ | $s17$ | $s18$ | $s19$ | $s20$ | $s21$ | $s22$ | $s23$ | $s24$ | $s25$ | y | |
|------|---------------------|------|------|------|------|------|------|------|------|------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-----|------|
| 3 | [3,3,3] | | | | | | | | | | | | | | | | | | | | | | | | | | 1 | |
| 3 | [3,3,4] | | | | | | × | | | | | | | | | | | | | | | | × | | | | | 1 |
| 3 | [3,3,4] | | | | | | | | | | | | | | | | | | | | | | × | | | | | 1 |
| 3 | [4,5,6] | | | | | | | | | | | | | | | | | | × | | | | | | | | | 1 |
| 4 | [3,3,3,4] | | | | | | | | | | | | | | | | | | × | | | | | | | | | 1 |
| 4 | [3,3,4,4] | | | | | | | | | | | | | | | | | | | | | | | | × | | | 1 |
| 5 | [3,3,3,3,5] | | | | | | | | | | | | | | | | | | | | | | | | × | | | 1 |
| 3 | [3,3,4] | | | | | | × | | | | | | | | | | | | | | | | | | | × | | 0.5 |
| 3 | [3,3,6] | | | | | | | | | | | | × | | | | | | | | | | | × | | | | 0.5 |
| 3 | [3,4,5] | | | | | | | | | | | | | | | | | | | | × | | | | | | × | 0.5 |
| 4 | [3,3,3,4] | | | | | | | | | | | | | | | | × | | | | | × | | | | | | 0.5 |
| 4 | [3,3,4,4] | | | | | | | | | | | | | | | | | | | | | | | | | | | 0.5 |
| 5 | [3,3,3,3,4] | | | | | | | | | | | | | | | | | × | | | | | | | | | | 0.5 |
| 3 | [3,4,5] | × | | | | | | | | | × | | | | | | | | | | | × | | × | | | | 0.25 |
| 4 | [3,3,3,4] | | | | | | | | × | | × | | | | | | | | | | × | | | | × | | | 0.25 |
| 4 | [3,3,3,4] | | | | | | | | | × | × | | | | | × | | | | | × | | | | | | | 0.25 |
| 4 | [3,3,4,5] | | | | | | | | × | | | | | | | | × | | | | | × | | | | × | | 0.25 |
| 3 | [3,3,3] | | | | | | | | | | | | | | | | × | | | | | | | | × | × | × | 0.2 |
| 3 | [3,3,5] | | | | | | | | | | | | | | | | × | | | | × | | | | × | × | × | 0.2 |
| 4 | [3,3,3,3] | × | | | | | | | | | | × | | | | | | | | | | × | | | | | | 0.2 |
| 4 | [3,3,4,5] | × | | | × | | | | | | | | | | | | | | | | | × | | | | × | × | 0.2 |
| 5 | [3,3,3,3,4] | | | | | | | | | | | × | | | | | | | | | | | × | × | | × | | 0.2 |
| 3 | [3,3,5] | | | | | | | | | | | × | × | × | | | | | | | × | | × | | | | | 0.16 |
| 5 | [3,3,4,4,5] | | | | | | | | | | | | | | | | | | | | | | | × | × | | × | 0.16 |
| 4 | [3,3,4,4] | | | | | | | | | | | | | × | × | | | | | | | × | × | | × | | | 0.14 |
| 5 | [3,3,3,4,4] | × | × | | | | | | | | | | | | | | | | | | | | × | × | | | × | 0.14 |
| 3 | [3,3,4] | | | | | | | | | | | | | | | | | | | | | | | | | | × | 0.11 |
| 6 | [3,3,3,3,3,5] | × | × | × | | | | | | | | | × | | | | | | | | | | × | × | | | | 0.11 |
| 3 | [3,4,4] | | | | | | | | | | | | | | | | | | | | | | | | | | × | 0.09 |
| 3 | [3,4,7] | | | | | | | | | | | | | | | | | | | | | | | | | | × | 0.08 |
| 3 | [3,3,3] | × | × | | | | | | | | | | | | | | | | | | | | | | | | × | 0.07 |
| 5 | [3,3,3,5,5] | | | | | | | | | | | | | | | | | | | | | | | | | | × | 0.07 |
| 4 | [3,3,3,3] | | | | | | | | | | | | | | | | | | | | | | | | | | × | 0.07 |
| 5 | [3,3,3,3,4] | × | × | | | | | | | | | | | | | | | | | | | | | | | | × | 0.07 |
| 5 | [3,3,4,4,4] | × | × | | | | | | | | | | | | | | | | | | | | | | | | × | 0.05 |
| 7 | [3,3,3,3,3,3,6] | × | × | | | | | | | | | | | | | | | | | | | | | | | | × | 0.04 |
| 3 | [4,5,5] | × | × | × | × | × | | | | | | | | | | | | | | | | | | | | | × | 0.04 |
| | FLUENCY | 10 | 10 | 5 | 10 | 6 | 9 | 10 | 6 | 7 | 11 | 8 | 7 | 8 | 7 | 8 | 9 | 10 | 11 | 12 | 16 | 9 | 14 | 15 | 11 | 16 | | |
| | ORIGINALITY | 1.2 | 1.1 | 0.6 | 1.4 | 0.9 | 2.2 | 1.2 | 0.7 | 0.9 | 1.4 | 0.8 | 1.1 | 0.7 | 0.7 | 0.8 | 1.5 | 3.3 | 0.9 | 2 | 2.5 | 1.8 | 2.7 | 3.7 | 1.7 | 2.1 | | |

($y = 2.1$) in originality (35% lower than the top y score). On the other hand, generating very few ideas does prevent original ideas as shown by the subjects with few responses ($f \leq 8$), all ranked in the lowest originality scores ($y \leq 0.79$, 50% lower than the average score). Looking at the *average* respondents ($f = 10$), the inadequacy of fluency to account for originality becomes more evident: subjects s1, s2, s4, s7 and s17, all generated the same number of solutions ($f = 10$) but they are very dissimilar in terms of original solutions; while s1, s2, s4 and s7 are some of the least original in the whole group (mean $y = 1.24$, 20% below average), s17 is the second highest in originality ($y = 3.3$). These results show that: a) individual results can be expected to vary significantly in a group, yet a number of common ideas easily found by a large majority can be expected, b) individuals with above-average fluency do show above-average originality, c) the most prolific individuals may fail to reach above-average originality, yet d) high originality breeds in above-average fluency.

This *ShapeStorm* study also helps to understand a critical aspect of divergent reasoning, *creative productivity*, the idea that individuals (or groups) can be evaluated by their ability to maximise original ideas while minimising total number of ideas. This skill has a clear advantage particularly when a design team transitions from idea generation to idea evaluation and selection, and is contrary to the generalised belief that more ideas are better, since by definition selecting the best ideas out of a larger pool adds significant difficulty and cost in stages of idea convergence. In this study, if fluency is taken as the main criterion, participants s20 and s25 would rank highest. If originality is taken as a top priority, participants s23 and s17 would come first. However, if productivity is prioritised based on the previous reasoning, then the top five performers include participant s6, who displays very high originality despite having a near average fluency. Moreover, beyond statistical analysis, and assuming that a creative person is defined by their ability to consistently generate very original ideas (not just good, but great ideas), one may look for participants who generated two or more highly original shape compositions, i.e., found by no-one else or less than 10% of the participants. The top three performers according to this view, are participants s17 (4 original ideas), s23 (3 original ideas) and s6 (2 original ideas). One may complement this analysis by observing how many common or obvious ideas ($y \approx 0$) these participants generated. With this in mind, participant s6 stands out with only two common ideas, while s17 generates four and s23 six common ideas. The ability displayed by s6 to generate highly original ideas in a small set of ideas shows high creative productivity that deserves further analysis.

Even such a simple *ShapeStorm* study helps understand the challenges of creative ideation research: even with outcomes that are objectively evaluated, results are open to interpretation and can be used to justify different evidence-based decisions depending on the criteria and intention of the exercise. In a

realistic setting where these results would be used to hire or promote candidates, or as the basis for team formation or selecting a winner in a competition, the results would vary depending on the views and assumptions of the decision makers. This also highlights the risk and limitations of the generalised practice in ideation research to rely on aggregate or average values, and the lack of validity in classifying people by their performance in standardised tests. This simple exercise shows that the underlying ideation process may be as or more important than the products achieved in a short session where a range of situational factors including chance can affect the outcome. Future extensions include systematic studies of the reasoning and strategy formation of participants as they solve this task using think-aloud research methods.

These illustrative results demonstrate the applicability and clarity of *ShapeStorm*, and provide an estimate of the type of results that one may find related to ideation fluency and originality. This is a well-defined simple yet rich visual task, language independent, where it is possible to objectively evaluate the results. The results shown here are consistent with our experience running this exercise with hundreds of high-school, undergraduate and graduate students in recent years. Nonetheless, these laboratory experiments cannot tell us alone how many valid and unique shape combinations exist in total even for the simplest version of *ShapeStorm* ($2g:3s$). In this exercise, 25 individuals found 37 unique solutions, but in average each individual generated less than 10. Over the years we had occasionally observed new solutions including some quite unusual ones, so we explore this question next via algorithmic approaches to *ShapeStorm* to illustrate how computers may help tackle this type of visual divergent tasks.

2.2 Stochastic ShapeStorm

This subsection explores the computational implementation of *ShapeStorm* using an algorithmic approach. The system presented here was initially inspired by our own observation of how participants represent the problem in hand-drawn *ShapeStorm*. It is reasonable to expect that a large number of ways of implementing the task are possible, which suggests future studies of divergence in algorithmic strategies not unlike the multiple solution tasks used in mathematical creativity (Leikin, 2009). For instance, parametric shapes can be iteratively rotated and scaled starting from an overlapping state.¹

In the method presented here, polygons are positioned in a two-dimensional coordinate space where the initial geometries are formed from arrays of $g \cdot s$ coordinate pairs, where g = number of initial geometries and s = number of sides, as previously explained in the description of *ShapeStorm*. This method falls in the category of random brute-force search strategies. As a first step, s points are randomly defined in a two-dimensional grid of arbitrary size. A

closed multiline is formed with these points. This is repeated g times, storing all initial polygons in an array G , and all initial multilines in an array L . As a second step, every line segment of every geometry in G , is checked for intersections with all other line segments in the array. When an intersection is detected, the line is subdivided and L is updated to replace the two intersecting lines with the line segments formed by the intersection. As a third step, using all lines in L , shapes are re-generated by building a shortest path tree (Dijkstra, 1959) with the resulting shapes stored in an array G' , and all duplicate entries are eliminated. If the resulting shape combination is valid according to the task description (i.e., the number of new shapes is greater than g), then a topological evaluation is carried using the geometries in G to check for: vertices of each initial geometry that fall within the area of other initial geometries but are not coincident with their line segments or vertices (i), vertices of each initial geometry that are coincident with the line segments of other initial geometries (l), or with their vertices (v), and all other vertices (o).

Figure 4 shows the shape combination shown in Figure 2a coded as $\{3,1,0,0,5(3,4,6)\}$, demonstrating the five representations described: 2D shapes in Figure 4a, array of polygons created by multilines in Figure 4b, the initial line segments in Figure 4c, the modified array of line segments partitioned by intersections in Figure 4d, and the topological coding in Figure 4e. This simple 'generate and test' process can be iterated by generating sets of initial geometries with random locations, identifying intersections, regenerating shapes with the new line segments, and evaluating the resulting composition for validity as well as originality or uniqueness.

All valid solutions are recorded during a simulation run, and a frequency metric is calculated in the end based on the number of instances generated for each solution. This random and memory-less process is called *exploration* and typically yields 11 unique solutions over one thousand steps, 19 solutions over ten thousand steps, around 28 solutions over one hundred thousand steps, and 33 solutions over one million steps – average of 30 runs in a 100×100 grid for *ShapeStorm* case ($2g:3s$). A sample set of 33 solutions generated with this stochastic method is shown in Table 2. The first column in Table 2 lists shape compositions using the notation previously introduced, which are sorted by their frequency or the number of instances in the second column. As a reference, the third column includes the frequency of these compositions in the hand-drawn exercise discussed above.

Whilst this strategy fails to generate all the solutions produced by humans, it does find valid solutions not included in the set of 245 solutions produced by the group of participants in our study. Some solutions align closely in the algorithmic and human processes such as $\{3,1,0,0,5 [3, 4, 6]\}$ (Figure 2a), which appears with high frequency in both systems, or $\{4,0,2,0,4 [3, 3, 3, 4]\}$ which is found by the algorithmic method only twice in one million runs, and only

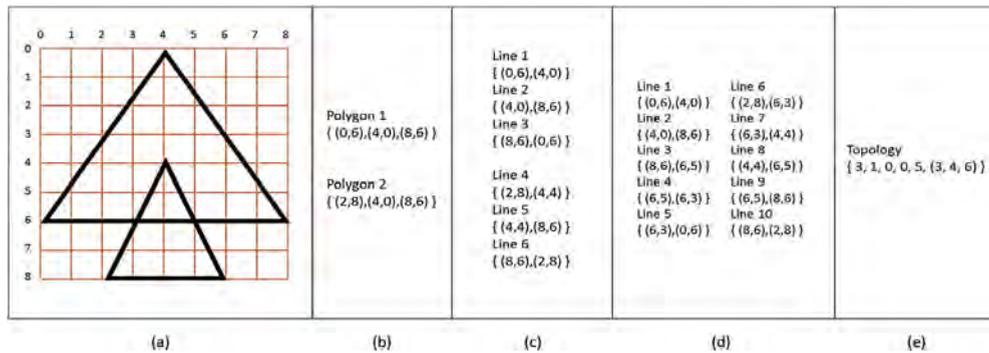


Figure 4 Solution coded as $\{3,1,0,0,5\{3,4,6\}$ is used to exemplify five ways of representing solutions: a) 2D shapes, b) array of polygons created by multilines, c) initial line segments, d) modified array of line segments partitioned by intersections, e) topological notation defined in Figure 2

Table 2 Sample set of 33 solutions generated with the stochastic method. First column lists shape compositions using the topological notation, sorted by frequency in the second column. Third column includes the frequency of these compositions in the hand-drawn exercise reported here

| Shape combinations | Solutions by computational exploration | Solutions by humans |
|-----------------------------------|--|---------------------|
| {5,0,0,0,6 [3, 3, 4, 4, 4]} | 253 598 | 76% |
| {3,1,0,0,5 [3, 4, 6]} | 160 226 | 96% |
| {5,1,0,0,5 [3, 3, 3, 5, 5]} | 94 688 | 52% |
| {3,2,0,0,4 [3, 4, 7]} | 49 013 | 48% |
| {5,1,0,0,5 [3, 3, 4, 4, 5]} | 28 237 | 24% |
| {3,2,0,0,4 [4, 5, 5]} | 23 081 | 92% |
| {7,0,0,0,6 [3, 3, 3, 3, 3, 3, 6]} | 12 171 | 84% |
| {5,2,0,0,4 [3, 3, 4, 4, 6]} | 4675 | 0% |
| {3,3,0,0,3 [4, 5, 6]} | 1958 | 4% |
| {4,0,1,0,5 [3, 3, 4, 4]} | 945 | 28% |
| {5,0,1,0,5 [3, 3, 3, 4, 4]} | 775 | 28% |
| {4,1,1,0,4 [3, 3, 4, 5]} | 395 | 20% |
| {3,1,1,0,4 [3, 4, 5]} | 345 | 16% |
| {6,0,1,0,5 [3, 3, 3, 3, 3, 5]} | 214 | 36% |
| {3,1,1,0,4 [3, 3, 6]} | 174 | 8% |
| {3,1,0,1,4 [3, 3, 5]} | 171 | 24% |
| {5,1,1,0,4 [3, 3, 3, 4, 5]} | 163 | 0% |
| {4,0,0,1,5 [3, 3, 3, 4]} | 150 | 16% |
| {5,0,0,1,5 [3, 3, 3, 3, 4]} | 121 | 12% |
| {4,1,1,0,4 [3, 4, 4, 4]} | 75 | 0% |
| {4,1,0,1,4 [3, 3, 4, 4]} | 68 | 4% |
| {3,2,1,0,3 [3, 4, 6]} | 61 | 0% |
| {3,2,1,0,3 [4, 4, 5]} | 23 | 0% |
| {4,2,1,0,3 [3, 4, 4, 5]} | 11 | 0% |
| {3,2,0,1,3 [4, 4, 4]} | 6 | 0% |
| {3,0,2,0,4 [3, 4, 4]} | 5 | 44% |
| {4,0,2,0,4 [3, 3, 4, 4]} | 3 | 8% |
| {5,0,2,0,4 [3, 3, 3, 3, 5]} | 3 | 4% |
| {4,0,1,1,4 [3, 3, 3, 3]} | 2 | 20% |
| {4,0,2,0,4 [3, 3, 3, 4]} | 2 | 4% |
| {3,1,1,1,3 [3, 3, 4]} | 1 | 0% |
| {4,1,2,0,3 [3, 3, 4, 5]} | 1 | 0% |
| {5,0,2,0,4 [3, 3, 3, 3, 4]} | 1 | 20% |

one human subject found it in the study. However, other combinations contradict these cases, such as $\{3,0,2,0,4 [3, 4,]\}$ which is highly unlikely in the algorithmic method described here ($<0.001\%$), but almost half of the participants in the study (44%) found it, or $\{5,2,0,0,4 [3, 3, 4, 4, 6]\}$ which is relatively easy to generate by our algorithm but no participant was able to spot in the study.

A detailed comparative analysis is beyond the purpose of this paper, it is likely that certain combinations are easier for humans due to symmetry or other gestalt principles, such as $\{7,0,0,0,6 [3, 3, 3, 3, 3, 6]\}$ (Figure 5a) which is infrequent in this algorithmic method but almost universal in our participants. The main finding of this computational implementation of *ShapeStorm* is that there seems to be strong evidence that humans engage in this visual divergent task partially following a random search, but this does not fully account for their generative process, which suggests that some heuristics are developed to guide the process.

An *exploitation* heuristic has been implemented, which builds on solutions found in an initial exploratory mode. This process works as follows: when previous solutions exist, one is chosen at random and its notation $\{i,l,v,o\}$ is used as a guide to modify the geometries in order to generate other new, valid and unique solutions. Figure 5 shows a concrete *exploitation* example: in Figure 5a, the shape combination of $(2g:3s)$ resulting in seven new shapes represented as $\{7,0,0,0,6 (3,3,3,3,3,3,6)\}$ is retrieved. A random vertex is selected from a random initial geometry from this composition and its location is shifted to a new position to test whether: a) the new arrangement conforms to the notation of the source combination, i.e. $(0,0,0,6)$, and b) it generates a new valid and original solution – in this case it does as $\{5,0,0,0,6 (3,3,4,4,4)\}$ as depicted in Figure 5b. The new position of the selected vertex is semi-randomly chosen from a set of points from the same region, namely inside the other shape (*i*), coincident with the perimeter of the other shape (*l*), coincident with the vertex (*v*), or outside the other shape (*o*). By using the topological relations derived from previously found solutions to guide the production of new solutions,

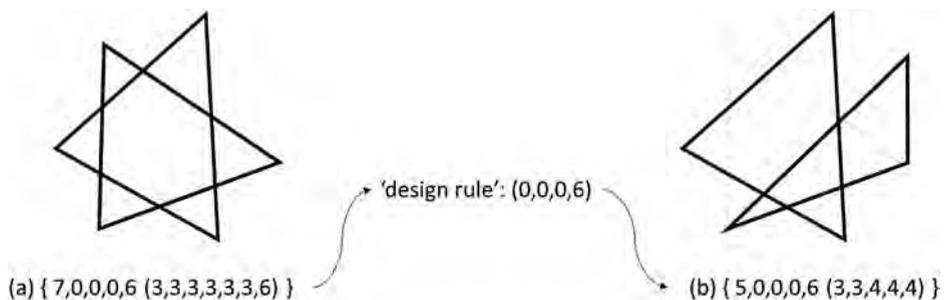


Figure 5 Exploitation example: a) solution $\{7,0,0,0,6 (3,3,3,3,3,3,6)\}$ is modified by selecting a random vertex and moving location to look for new valid and original solutions – in this case b) solution $\{5,0,0,0,6 (3,3,4,4,4)\}$

the algorithm can be said to be guided by its results – albeit the process remains essentially stochastic.

The results with this exploitation heuristic show a significant improvement, with 19 unique solutions in one thousand steps, 29 unique solutions in ten thousand steps, and 38 solutions in one hundred thousand steps for *ShapeStorm* case (2g:3s) – average of 30 runs in all cases. This effect of reducing the steps to generate results by one order of magnitude when using *exploitation* suggests promising experimental setups to study with human participants, such as supporting their search by a) priming them with solutions of different types, b) helping them build heuristics, and c) compare solutions generated by teammates, etc.

Such algorithmic approaches offer alternative models to reason about and possibly benchmark human performance in *ShapeStorm*. They rely heavily on randomness, even when *exploitation* heuristics are used. Because of the combinatorial explosion of the task, there is no guarantee that these systems would exhaust the space of solutions in reasonable times. So, while stochastic approaches do show that the solution space is considerably larger than in hand-drawn conditions, the question remains: what is the actual size of the solution space of *ShapeStorm* even for the simplest case (2g:3s)? Without such characterisation, we may derive only relative originality scores based on what humans or stochastic algorithms ever find, but it would be useful to estimate complete measures of originality or completeness for the task, which

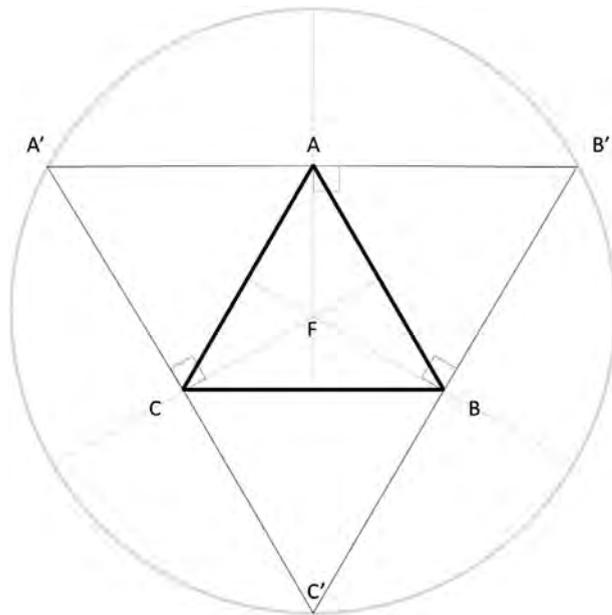


Figure 6 One initial triangle is defined as an equilateral triangle ABC fixed in the plane with geometric centroid F

was detected in the literature as a key limitation of the current practice of defining creativity tasks. The following section tackles this important question by presenting a computational geometry proof to exhaustively map the solution space.

3 ShapeStorm: *complete solution space for case (2g:3s)*

This section presents results of a geometrical analysis to compute the complete solution space of the simplest *ShapeStorm* case (2g:3s). The method is based on a computation of all the feasible possibilities from the topological viewpoint of the intersection of two triangles (Devillers & Guigue, 2002). We define this method as follows: first, define one of the initial triangles as a fixed area in the plane; in this case, without loss of generality, we demonstrate the method with an initial equilateral triangle ABC as shown in Figure 6. The second step is to draw the geometric centroid or centre of mass of the polygon vertices of this triangle, called F – which is also the intersection of the three triangle medians. Then, we draw lines through points A , B , and C , which are perpendicular to AF , BF , and CF , respectively. These lines meet to form yet another equilateral triangle, $A'B'C'$. The circumcircle of $A'B'C'$ is drawn using F as the centre.

Now, the polygon sides of the triangle ABC are extended in order to demarcate twelve regions (D_1 to D_{12}) as shown in Figure 7, where the vertices of the second triangle $A'B'C'$ are located. D_1 refers to the region covered by triangle ABC , D_2 to D_4 are the regions adjacent to the polygon sides of triangle ABC ; D_5 to D_7 are the boundary regions formed by the extended lines of triangle ABC ; finally, D_8 to D_{12} are the regions between the extended lines of triangle ABC and regions D_2 to D_4 . These regions define all possible locations for the vertices of the second geometry, identified as triangle DEF . Note this model is valid for any triangle ABC .

The vertices for the second triangle DEF are next defined by iterating positions through all of these twelve regions, D_1 to D_{12} . Figure 8 shows all the specific positions defined within each of the twelve regions and a sample instance of triangle DEF with vertices in D_1 , D_4 , and D_{12} . In order to capture all the topological combinations between the two geometries, multiple points in all regions are defined to avoid point coincidence in triangle DEF . As shown in Figure 8, three points in D_1 are defined in the vertices of triangle ABC ; three more points in D_1 are defined coincident with the polygon lines of triangle ABC in the midpoint and the intersections of lines connecting the midpoint of the opposite polygon line and the vertices of triangle $A'B'C'$; the geometric centroid of triangle ABC and the midpoints of the three triangle medians define the last four points in D_1 – a total of 16 points in this region. Two points are defined in D_2 to D_4 , the vertices of triangle $A'B'C'$ and the midpoints between these vertices and the midpoints of the corresponding side of triangle ABC . Five points are

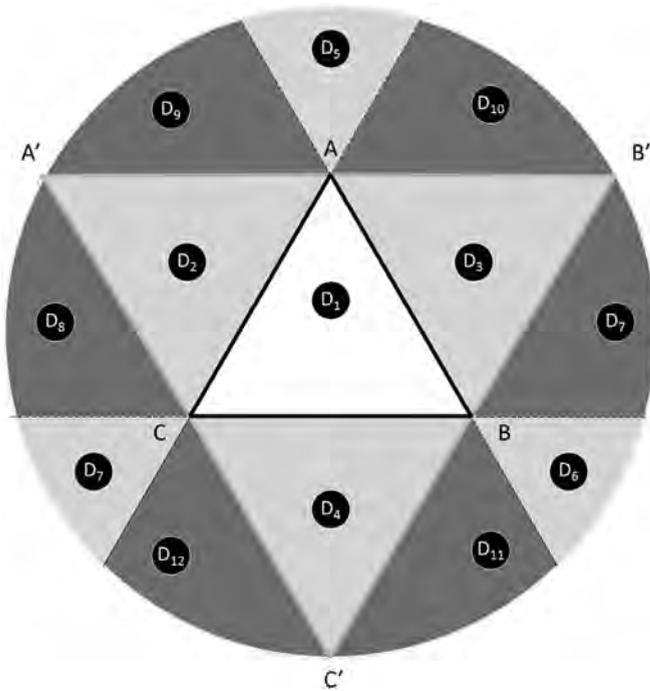


Figure 7 The polygon sides of the triangle ABC are extended in order to demarcate twelve regions (D_1 to D_{12}) where the vertices of the second triangle DEF will be located

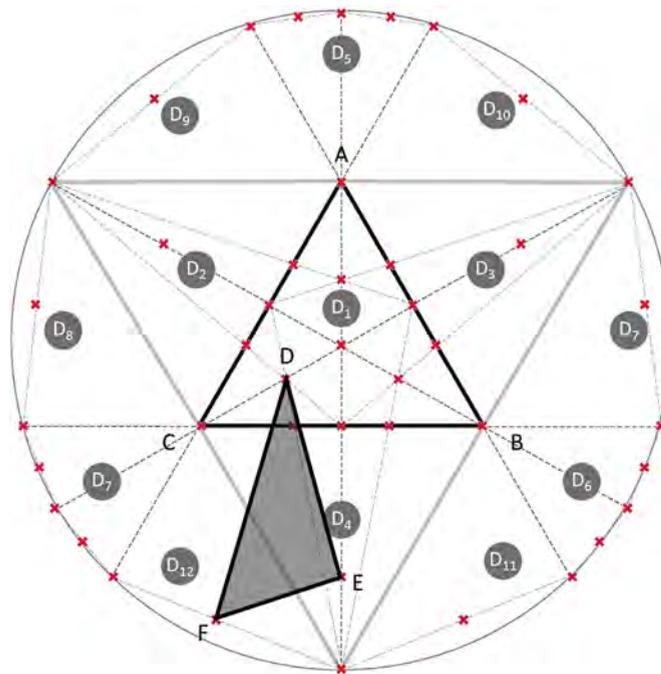
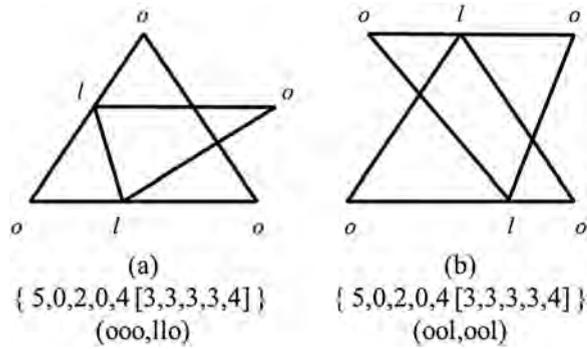


Figure 8 The vertices for triangle DEF are defined iterating positions through all twelve regions, D_1 to D_{12} . All the specific positions defined within each of the twelve regions are shown here and a sample DEF in D_1 , D_4 , and D_{12}

Figure 9 Sample shape combinations with shared topological notation $\{5,0,2,0,4 [3,3,3,3,4]\}$ are disambiguated using two strings of characters that identify the vertex positions for each shape in relation to its counterpart



defined in D_5 to D_7 , two in the intersection of the extended lines of triangle ABC with the circumcircle of $A'B'C'$, one in the intersection of the extended median of triangle ABC with the circumcircle of $A'B'C'$, and two in the midpoints between straight lines connecting these points. Finally, one point is defined in D_8 to D_{12} , at the midpoint of straight lines connecting the vertices of triangle $A'B'C'$ and the intersection of the extended lines of triangle ABC with the circumcircle of $A'B'C'$. In all, the permutation of these points is obtained by $P = 43!/(43 - 3)!$ with a total of 74 046 possible variations for triangle DEF .

With this setup, a deterministic search procedure is implemented computationally to generate and classify all permutations of points DEF keeping triangle ABC constant. An initial inspection of the results using the notation previously defined, namely $\{g',i,l,v,o, (s'_1... s'_n)\}$, shows this to be insufficient to distinguish between certain shape combinations, such as $\{5,0,2,0,4 [3,3,3,3,4]\}$, illustrated in Figure 9. To this end, the notation is refined to include a sequence of characters that identify the relative position for all polygon vertices in an ordered chain for the original geometries. As a result, solutions in Figure 9 are discriminated by the relative location of the vertices of the two triangles in *ShapeStorm* case (2g:3s): for Figure 10a the strings (ooo,llo) indicate that the three vertices of the first triangle lie outside the second triangle and for the second triangle, one lies outside and two are coincident with the lines of the first triangle; for Figure 9b, the strings (ool,ool) indicate that two vertices lie out and two are coincident with the lines of each triangle.

Accounting for all permutations in the method described previously, the results using this extended notation show a total of 59 unique solutions for *ShapeStorm* case (2g:3s). With this method, the complete solution space can be fully characterised, revealing its size and structure which can be used to model heuristics derived intuitively by human subjects, or to build

computational systems to teach or support visual divergence. Identifying the solution space of such visual divergence task also enables classification of solutions by the number of occurrences, by topological similarities, or by the set of regions where the vertices are located – directions beyond the scope of this paper. For reference, the extended notation for all shape compositions is shown in Table 3 – a visual representation of all solutions is omitted to avoid the use of *ShapeStorm* in accidentally priming ongoing and future studies.

Figure 10 shows two cases (5 and 6 in Table 3) that are similar, yet reveal qualitative differences in the construction process. Here, two shape combinations share the same notation ‘{3,0,2,2,2 [3,3,3]}’ or in plain English: three final triangles with no vertices from the first initial triangle within the area of the second initial triangle, two vertices coincident with the lines and one pair of coincident vertices between initial triangles, and two points outside – and each initial triangle represented by (vlo, vlo) : one vertex lying outside, one vertex coincident in line and one in the vertex of each other. However, as shown in the visual representation in Figure 10 showing the initial triangles shaded in yellow (light) and red (dark), two distinctive procedures are used to generate these combinations. The need to account for such differences is further confirmed by the fact that the combination in Figure 10a was found by 20% of the human participants in our study, while Figure 10b was not found by any of them.

The results of this geometric analysis help shed new light into the study presented in Section 2.1, where participants generated an average of 10 solutions individually. This is less than 20% of the complete solution space. 37 unique instances were found by all 25 participants *together*, which shows that almost 40% of the solutions remained inaccessible to the group. Interestingly, the stochastic model presented in Section 2.2 has a reach of about 38 solutions in simulations of one million steps using only random search. We did not expect to see such a gap between humans and computers, particularly with the *ShapeStorm* case ($2g:3s$). The stochastic model is scalable, so one can reasonably expect that three triangles or one triangle and one rectangle would produce even more significant gaps between human and computational systems.

Figure 10 Sample shape combinations with shared detailed topological notation {3,0,2,2,2 [3,3,3]} showing the need for further disambiguation left for future work

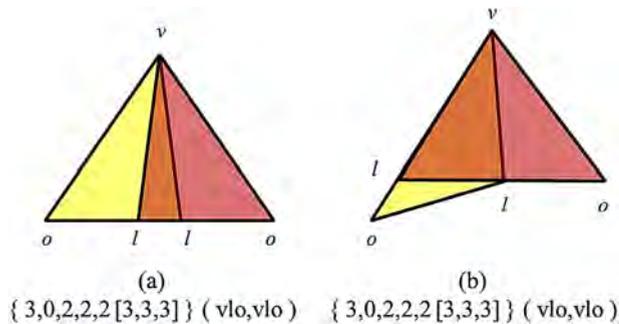


Table 3 Complete solution space of *ShapeStorm* (2g:3s)

| | | | |
|----|-----------------------------------|----|---|
| 1 | {3,0,0,4,2 [3, 3, 3]} ovv,ovv, | 31 | {4,0,1,0,5 [3, 3, 4, 4]} loo,ooo, |
| 2 | {3,0,1,2,3 [3, 3, 4]} lov,oov, | 32 | {4,0,1,2,3 [3, 3, 3, 3]} lov,oov, |
| 3 | {3,0,2,0,4 [3, 4, 4]} loo,loo, | 33 | {4,0,1,2,3 [3, 3, 3, 4]} lov,oov, |
| 4 | {3,0,2,2,2 [3, 3, 3]} llv,oov, | 34 | {4,0,2,0,4 [3, 3, 3, 4]} llo,ooo, |
| 5 | {3,0,2,2,2 [3, 3, 3]} lov,lov* | 35 | {4,0,2,0,4 [3, 3, 3, 4]} loo,loo, |
| 6 | {3,0,2,2,2 [3, 3, 3]} lov,lov* | 36 | {4,0,2,0,4 [3, 3, 4, 4]} ooo,llo, |
| 7 | {3,0,2,2,2 [3, 3, 3]} oov,llv, | 37 | {4,0,3,0,3 [3, 3, 3, 3]} lll,ooo, |
| 8 | {3,0,2,2,2 [3, 3, 4]} lov,lov, | 38 | {4,0,3,0,3 [3, 3, 3, 3]} llo,loo, |
| 9 | {3,0,3,0,3 [3, 3, 4]} llo,loo, | 39 | {4,0,3,0,3 [3, 3, 3, 4]} llo,loo, |
| 10 | {3,0,3,0,3 [3, 3, 4]} loo,llo | 40 | {4,1,0,2,3 [3, 3, 4, 4]} iov,oov, |
| 11 | {3,0,3,0,3 [3, 3, 4]} ooo,lli | 41 | {4,1,1,0,4 [3, 3, 4, 5]} ilo,ooo, |
| 12 | {3,1,0,0,5 [3, 4, 6]} ioo,ooo, | 42 | {4,1,1,0,4 [3, 3, 4, 5]} loo,ioo, |
| 13 | {3,1,0,2,3 [3, 3, 5]} iov,oov, | 43 | {4,1,1,0,4 [3, 4, 4, 4]} ilo,ooo, |
| 14 | {3,1,1,0,4 [3, 3, 6]} ilo,ooo, | 44 | {4,1,2,0,3 [3, 3, 4, 4]} ilo,loo, |
| 15 | {3,1,1,0,4 [3, 4, 5]} ioo,loo, | 45 | {4,1,2,0,3 [3, 3, 4, 4]} llo,ioo, |
| 16 | {3,1,1,2,2 [3, 3, 4]} ilv,oov, | 46 | {4,1,2,0,3 [3, 3, 4, 5]} ioo,llo, |
| 17 | {3,1,1,2,2 [3, 3, 4]} iov,lov, | 47 | {4,2,1,0,3 [3, 4, 4, 5]} ilo,ioo, |
| 18 | {3,1,1,2,2 [3, 4, 4]} iov,lov, | 48 | {5,0,0,0,6 [3, 3, 4, 4, 4]} ooo,ooo, |
| 19 | {3,1,2,0,3 [3, 3, 5]} ill,ooo, | 49 | {5,0,0,2,4 [3, 3, 3, 3, 4]} oov,oov, |
| 20 | {3,1,2,0,3 [3, 3, 5]} ilo,loo, | 50 | {5,0,1,0,5 [3, 3, 3, 4, 4]} loo,ooo, |
| 21 | {3,1,2,0,3 [3, 4, 4]} llo,ioo, | 51 | {5,0,2,0,4 [3, 3, 3, 3, 4]} llo,ooo, |
| 22 | {3,1,2,0,3 [3, 4, 5]} ilo,loo, | 52 | {5,0,2,0,4 [3, 3, 3, 3, 4]} loo,loo, |
| 23 | {3,1,2,0,3 [3, 4, 4]} ooo,lli | 53 | {5,0,2,0,4 [3, 3, 3, 3, 5]} loo,loo, |
| 24 | {3,2,0,0,4 [3, 4, 7]} iio,ooo, | 54 | {5,1,0,0,5 [3, 3, 3, 5, 5]} ioo,ooo, |
| 25 | {3,2,0,0,4 [4, 5, 5]} ioo,ioo, | 55 | {5,1,0,0,5 [3, 3, 4, 4, 5]} ioo,ooo, |
| 26 | {3,2,0,2,2 [4, 4, 4]} iov,iov, | 56 | {5,1,1,0,4 [3, 3, 3, 4, 5]} ioo,loo, |
| 27 | {3,2,1,0,3 [3, 4, 6]} iio,loo, | 57 | {5,2,0,0,4 [3, 3, 4, 4, 6]} ioo,ioo, |
| 28 | {3,2,1,0,3 [4, 4, 5]} ilo,ioo, | 58 | {6,0,1,0,5 [3, 3, 3, 3, 3, 5]} loo,ooo, |
| 29 | {3,3,0,0,3 [4, 5, 6]} iio,ioo, | 59 | {7,0,0,0,6 [3, 3, 3, 3, 3, 6]} ooo,ooo, |
| 30 | {4,0,0,2,4 [3, 3, 3, 4]} oov,oov, | | |

4 Discussion

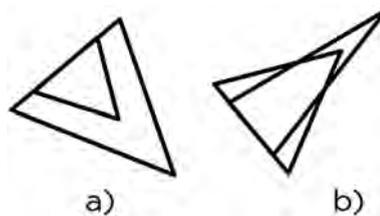
Divergent visual reasoning is a fundamental skill in design creativity, but rigorous modelling options are missing to date. The model *ShapeStorm* is motivated by the challenge of building the criteria, guidelines and taxonomy of robust creativity tasks that can be used across design studies including humans, computers, and hybrid systems. This model emerged from observing the performance of humans in a drawing exercise that stimulates divergent reasoning and evolved to its current version by comparing their results with those generated by stochastic algorithms and ultimately with the complete solution space obtained by geometrical analysis.

The usefulness of a model such as *ShapeStorm* is demonstrated by its capacity to capture a wide range of issues connected to the assessment of creativity, the underlying assumptions in the choice of indicators, and the type of insights built from interpreting the creative fluency of idea generators. In ideation studies, ideas are customarily tallied and evaluated, and the participants or their processes are ranked based on these scores. The hand-drawn version of

ShapeStorm illustrates the intricacies behind the moderate correlation – unchallenged in the literature – between fluency and originality, as discussed at length in section 2.1. It yields concrete evidence that individual variability in quantitative and qualitative indicators can be expected to be high, and that while many individuals with above-average fluency show above-average originality, the most prolific individuals may fail to reach above-average originality, and conversely the individuals with the most original ideas may not be the most prolific, albeit originality does seem to breed in above-average fluency.

In addition to articulating such limitations in the evaluation of creative ideas, preliminary versions of *ShapeStorm* have been used in computational studies of group ideation (Sosa & Gero, 2012a), idea incubation principles (Sosa & Gero, 2012b), and to reason about the creative value of bad ideas (Sosa & Gero, 2013). In the last case, it has been instrumental to illustrate how certain types of ‘bad ideas’ (low performance) can be highly valuable for creative ideation when they serve as means to access uncommon ideas of high performance. Figure 11 illustrates this point in *ShapeStorm* showing that low-score or even invalid solutions may lead to valuable and uncommon solutions via small variations. In a customised task where the goal is to find three or more final geometries including shapes of five sides, combination $\{2,1,2,0,3\}$ (3,6) depicted in Figure 11a would receive a very low creativity score since it only produces two final geometries of three and six sides. Nonetheless, a change in the location of a single point in $\{2,1,2,0,3\}$ (a minimal ‘design move’) leads to solution $\{4,1,2,0,3\}$ (3,3,4,5) as depicted in Figure 11b. This resulting solution not only receives a high score given the evaluation criteria of the task, it also seems to be rather uncommon as suggested by the results of the study presented in Section 2.1 where no participants found it. This combination is also highly improbable using the stochastic algorithm presented in Section 2.2, and has one of the lowest number of instances in the analysis presented in Section 3. These metrics show that $\{4,1,2,0,3\}$ (3,3,4,5) has a low *accessibility* score in *ShapeStorm* and for this reason, the ‘bad’ or illegal combination $\{2,1,2,0,3\}$ (3,6) is a good example of the ‘creative value’ of bad ideas, i.e., the high potential of certain poor ideas to lead to great ideas

Figure 11 A change in the location of a single point in a) solution $\{2,1,2,0,3\}$ (3,6) leads to b) solution $\{4,1,2,0,3\}$ (3,3,4,5)



due to their close connectedness despite their distance in the fitness landscape. This principle captured in *ShapeStorming* is reminiscent of research on the potential value of bad ideas in problem solving supporting ideation techniques where low-performing ideas are viewed as ‘positive detours enlarging the pool of ideas’ (Dix & Gongora, 2011).

The main significance of *ShapeStorm* is that it offers a straightforward means to define a simple design task that can be used across research studies. As demonstrated in this paper, *ShapeStorm* is a task with simple and clear instructions, based on visual representations and suitable for wide age ranges, literacy levels and cultures. It is suitable for short activities such as lab or classroom sessions. It has objective evaluation metrics, eliminating reliance on experts to assess the creativeness of output, and supports additional assessment criteria and task constraints. Since all possible solutions are known at least for one of its simplest cases, reliable measures can be established avoiding fallible assessment situations. Having established the characteristics of the entire solution space for such cases, *ShapeStorm* makes it possible to estimate relative and universal scores of visual divergence. This task certainly does not resolve all the challenging aspects of measuring creative ideation, but we hope it contributes to the multi-faceted enterprise of understanding human and computational creativity.

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